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6. AUTHORS Negash G. Medhin			
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**CLARK ATLANTA UNIVERSITY**

**CONTROL AND COMPUTATION IN SMART MATERIAL  
STRUCTURES**

**Final report on research done under  
Grant No. F49620-97-0495**

**For The Air Force Office of Scientific Research  
Bolling AFB, DC**

**By**

**Clark Atlanta University,  
James P. Brawley Dr.  
Atlanta, Georgia**

**P.I. : Negash G. Medhin**

**September 30, 1998**

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## CONTENTS

1. Summary	Page 1
2. Mathematical Modeling, Analysis of a Laminated Beam Embedded Piezoceramic Patches	Page 1
3. Analytical and Numerical Consideration	Page 1
4. Design of Control	Page 2
5. Identification Problem	Page 2
6. Conclusion	Page 2
7. Publications	Page 3
8. Presentations	Page 3
9. Professional Travel	Page 3
10. M.S. Thesis Supervised	Page 4

## **Control and Computation in Smart Material Structures**

### **1. Summary**

The research carried out dealt with mathematical modeling, computation, and model validation in smart material structures.

In particular a focused study of a laminated curved beam with embedded piezoceramic patches was conducted. There has been widespread interest in the engineering and industrial community on the use of composite structures for acoustic application, and other complex articulated structures. However, mathematical foundation for these devices is not adequate. Our research contributes toward filling this gap.

### **2. Mathematical Modeling and Analysis of a Laminated Beam with Embedded Piezoceramic Patches.**

As is well known elastic structural members have excessive vibration at resonant frequencies. By bonding a viscoelastic layer covered by constraining layer one can achieve high damping. This is what is called constrained layer damping. More recently there is a growing interest in active constrained layer(ACL) damping. This is often effected in some type of smart or adaptive material configuration using sensor/actuator devices such as patch layers with appropriate circuitry and power supply. Typically these models involve linear elasticity(parent structure), viscoelastic material and piezoceramic actuators and sensors.

In [BMZ1] we considered a laminated curved beam with an elastic core sandwiched between symmetrically bonded viscoelastic layers. The resulting structure is sandwiched between symmetrically bonded constraining layers. Then, piezo patches are bonded symmetrically on either side of the curved composite structure. Hysteresis is present in active sensor/actuator components as well as passive components of smart material structures. A linear integral hysteresis is assumed in the viscoelastic components. Shear effect in the viscoelastic components is also accounted for. The main goal in this effort is to obtain an accurate mathematical model that is well-posed. Further, a rigorously justified approximation procedure is included.

### **3. Analytical and Numerical Considerations**

In the above model the piezo patches provided unbounded inputs. The dynamical equation of the mathematical model is an abstract Cauchy problem. The right hand side of the governing equation contains the Dirac measure and its derivatives. In order to accommodate these generalized functions it is necessary to enlarge the space in which the

abstract Cauchy equation is considered. In the enlarged space we also justify Galerkin approximation. Details of this work are presented in [BBM].

#### **4.Design of Control**

Here a methodology to design control for the purpose of suppressing vibration using the piezo patches is considered. Simultaneously one can also investigate the role of viscoelasticity in damping the vibration.

The abstract Cauchy problem governing the vibration of the beam is approximated using B-splines and cubic B-splines. Then, the weak form of the infinite dimensional problem is restricted to the finite dimensional approximating space. Then, a quadratic cost criterion is minimized. The minimization problem characterizes the level of voltages that are applied to the piezo patches as a function of time in order to suppress the vibration of the curved viscoelastic beam. Complete details are found in [BM].

#### **5.Identification Problem**

To use piezoelectric actuators effectively the type and location of the actuators and the applied voltages corresponds to the mathematical problem of identifying a piecewise smooth function. A direct and practical approach to this problem is to pose a parameter estimation problem and get an approximation. In the work completed a completely analytic approach is carried out. It is shown analytically how to recover the unknown function from boundary observation under certain mathematical assumptions. The work completed is not meant to be a complete solution to the problem. The reason for this is that the mathematical assumptions are not completely realistic. A completely realistic model requires further work and more refined mathematical tools. However the basic ideas remain the same. Complete details are presented in [AM].

#### **6.Conclusion**

Physically based accurate mathematical models of laminated composite beams with embedded piezo patches are presented. The model equations are shown to be well posed and a rigorously justified approximation framework is also presented.

Design of control was investigated and computational procedure presented. Analytical approach to an associated identification problem is completed.

From the literature, and our effort to date we have observed lack of reliable and satisfying numerical codes. The problems considered all lead to large size and difficult problems requiring enough time and a deeper understanding of the numerics.

## **7.Publications**

- [AM] S.A. Avdonin, N.G. Medhin, Identification of Piecewise Constant Coefficient in Beam Equation( To appear in Jour. Of Computational & Applied Mathematics-special issue.)
- [MBZ] H.T. Banks, N.G. Medhin, Y. Zhang, Mathematical Model for a Laminated Curved Beam, Proc. 36th Conf. In Decision & Control, pp. 3739-3748, 1997.
- [MBZ1] H.T. Banks, N.G. Medhin, Y. Zhang, Mathematical Model and Analysis for a Laminated Curved Beam with Shear, Dynamic Systems and Applic, Vol 7, No. 3, pp. 291-318, 1998.
- [BBM] H.T. Banks, N. Begashaw, N. Medhin, Analytical and Numerical Treatment of a Curved Constrained Layer Structure(to be submitted.)
- [BM] N. Begashaw, N. Medhin, Design of Control in Smart Material Structures(to be submitted.)
- [MS] M. Sambandham, N. Medhin, An Extremum Problem with Application to Vibration Suppression in a Beam( to be submitted.)

## **8.Presentations**

1. Mathematical Model for a Laminated Curved Beam, 36th IEEE Conf. In Decision and Control, December, 1997, San Diego, California.
2. Approximation Methodology for a Volterra Integrodifferential Equations with Applications to Curved Beams, AMS, Jan. 7-10, Baltimore, Maryland.
3. Identification of Piecewise Constant Coefficient of a Beam via Boundary Observation, 4<sup>th</sup> SIAM Conf. On Control and Its Applications, May 7-9, 1998, Jacksonville, Florida. The same material was also presented at Georgia Institute of Technology.

## **9.Professional Travel**

- October 1997, North Carolina State University
- October 1997, Lord Corporation, Cary, North Carolina
- April 1998, North Carolina State University
- July 1998 North Carolina State University

**10. M.S. Thesis Supervised**

Roslynn Bryant: An Analysis of an Equation Governing The Transverse Displacement of a Beam, May 1998.